Note: As students graduate to more complex algebraic procedures, such as solving problems containing exponents and factoring polynomials, they may still rely on the same learning strategies that helped them understand the basics of algebra. This follow-up set of case studies takes the strategies introduced in Algebra (Part 1)—teaching through math vocabulary, graphic organizers, Concrete-Representational-Abstract method, and mnemonic devices—and illustrates how to apply them to more advanced concepts.

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To cite this case study unit:


For an Instructor’s Guide to this case study unit, please email your full name, title, and institutional affiliation to the IRIS Center at iris@vanderbilt.edu.

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Algebra Introduction

General Guidelines for Teaching Algebra

Many students, including those with disabilities, have difficulty with the abstract concepts taught in algebra. The following guidelines should be considered when teaching algebra:

- **Curriculum** – Often times, the curriculum adopted by school districts moves too quickly and students may not fully understand the current concept before they are forced to move on to the next concept. It is important that teachers provide students with ample time to learn the concept and a sufficient number of opportunities to practice the concept.

- **Language of mathematics** – It is important that students are able to define and use algebra terminology. A list of important vocabulary terms and strategies for teaching them are included in this case study set.

- **Prerequisite skills** – Students must have knowledge of prerequisite skills prior to learning algebra. These skills may include but are not limited to basic facts, problem solving skills, and probability skills. It may be necessary to review these skills prior to working with algebra concepts.

- **Modeling by teacher** – Teachers must model strategies prior to allowing students to complete work on their own. During modeling, teachers talk aloud as they demonstrate how to solve a problem. They should model error and self-correction strategies. This should be continued until enough problems have been modeled so that students understand the concept and/or how to use the manipulatives provided.

- **Real-life examples** – It is imperative that algebra problems be related to real-life situations. Students often ask why algebra is necessary; relating it to real-life situations will encourage the connection. It is also important for students to have strategies for deciding how to set up the problems they need to solve.

- **Effective instruction** – Teachers must make certain they understand algebra well enough to teach it to their students. Effective teaching behaviors (e.g., specific praise, questioning) should also be included in all lessons.

- **Error analysis** – Error analysis is the process of looking closely at student errors to determine what they are doing incorrectly. Error analysis can be done by examining the problems or by interviewing students and asking them to demonstrate what they have done.

- **Reviewing material** – It is important that students receive ample opportunities to review what has previously been learned, in order for them to maintain the knowledge.

- **Calculators** – The use of calculators will assist students in completing complex math problems, including algebra. The use of calculators in algebra is complex, and students will need explicit instruction on how to use the calculators.

- **Concrete materials** – The use of concrete materials or manipulatives will assist students in understanding the abstract level of algebra. These manipulatives may include algebra tiles, algeblocks, or other items.

- **Promoting a positive attitude toward math** – Teachers must show enthusiasm when teaching algebra.
Resources...


BACKGROUND
Student: Luke
Age: 14.6
Grade: 9th
Focus: Inequalities and absolute value

SCENARIO
It is January and Luke’s class has moved onto the concepts of inequalities and absolute value in algebra. Luke is having a great deal of difficulty with these concepts. His teacher feels she is moving too quickly for Luke but does not feel she can slow down, as the rest of the class understands the concepts. She has arranged for Luke to work with a peer to help him achieve the following goals:

• Given inequalities that involve addition, subtraction, multiplication, and division (e.g., \( h + 15 \geq 5 \); \( m - 2.2 < 12.2 \); \( 9x > 18 \)), Luke will solve.
• Given compound inequalities (e.g., \( 2 < x < 5 \)), Luke will solve.
• Given absolute value equations (e.g., \( |3x - 2| = 10 \)), Luke will solve.

POSSIBLE STRATEGIES
• Teaching Vocabulary
• Graphic Organizers

ASSIGNMENT
1. Read the General Guidelines for Teaching Algebra provided at the beginning of this case study.
2. Read the STAR Sheets for each possible strategy listed above.
3. Summarize the components of each strategy. Be sure to include how each strategy will support Luke and the benefits of using each strategy.
4. Using one or more of these strategies, describe an activity that could be used to assist Luke.
BACKGROUND

Student: Jamal
Age: 14.6
Grade: 9th
Focus: Exponents and exponential functions

SCENARIO

Jamal’s algebra class has begun working with exponents. Jamal understands how to solve for basic exponents (e.g., $4^6$) but is having difficulty with more advanced exponent problems. Jamal’s teacher is willing to implement strategies to assist Jamal in achieving his goals, which include:

- Given exponent problems (e.g., $4^6$; $(5c^5)(-b^8c^2)$; $10^4 - 10^2$), Jamal will simplify.
- Given problems that require simplifying quotients of powers (e.g., $2^7 \div 2^4$), Jamal will simplify.
- Given exponential function problems, Jamal will solve.

POSSIBLE STRATEGIES

- Teaching Vocabulary
- Graphic Organizers

ASSIGNMENT

1. Read the General Guidelines for Teaching Algebra provided at the beginning of this case study.
2. Read the STAR Sheets for each possible strategy listed above.
3. Summarize the components of each strategy. Be sure to include how each strategy will support Jamal and the benefits of using each strategy.
4. Using one or more of these strategies, describe an activity that could be used to assist Jamal.
BACKGROUND
Student: LaTanya
Age: 14.6
Grade: 9th
Focus: Polynomials and factoring

SCENARIO
LaTanya’s class has begun working on polynomials and factoring. LaTanya had no difficulty when working with expressions at the beginning of the algebra course, but she is now having difficulty understanding the concepts presented. After working with LaTanya, her teacher realizes she is having difficulty with the many steps required to solve these problems. LaTanya’s teacher has decided to implement some strategies during her class in order to assist LaTanya, and other students, in remembering the process needed to solve the problems and to meet the following goals:

• Given polynomial problems (e.g., (x + 4) (x - 2); (2x + 2) + (3x - 3)), LaTanya will simplify.
• Given equations (e.g., 4x^2 + 12; x^2 + 4x + 10), LaTanya will factor.

POSSIBLE STRATEGIES
• Teaching Vocabulary
• Concrete-Representational-Abstract Method
• Mnemonic Devices

ASSIGNMENT
1. Read the General Guidelines for Teaching Algebra provided at the beginning of this case study.
2. Read the STAR Sheets for each possible strategy listed above.
3. Summarize the components of each strategy. Be sure to include how each strategy will support LaTanya and the benefits of using each strategy.
BACKGROUND
Student: Tyler
Age: 14.5
Grade: 9th

SCENARIO
It is the beginning of the second semester and Tyler is having a great deal of difficulty in his algebra class. He has an understanding of the basic concepts of algebra but has not mastered the skills needed to move to the higher-level skills that his class is working on. Tyler’s teacher has spoken with his parents about the possible need for additional support and his parents have agreed to help at home. The following are goals for Tyler to achieve:

• Given exponent problems (e.g., $4^6$; $(5c^3)(-b^8c^2)$; $10^4 - 10^2$), Tyler will solve.
• Given problems that require simplifying quotients of powers (e.g., $2^7 ÷ 2^4$), Tyler will solve.
• Given compound inequalities (e.g., $2 < x < 5$), Tyler will solve.
• Given inequalities that involve addition, subtraction, multiplication, and division (e.g., $h + 15 ≥ 5$; $m - 2.2 < 12.2$; $9x > 18$), Tyler will solve.
• Given absolute value equations (e.g., $|3x - 2| = 10$), Tyler will solve.

POSSIBLE STRATEGIES
• Teaching Vocabulary
• Graphic Organizers
• Concrete-Representational-Abstract Method
• Mnemonic Devices

Assignment
1. Read the STAR Sheets for the four strategies listed above.
2. Sequence Tyler’s goals in the order you would address them.
3. For each goal, identify a strategy and explain how it will assist Tyler in reaching his goals.
BACKGROUND
Student: Jeffery
Age: 13.9
Grade: 9th

SCENARIO
Jeffery has been doing very well in algebra. He understands the basic concepts of algebra and enjoys solving the problems. However, Jeffery is having difficulty with the concepts requiring higher-level reasoning skills. His specific areas of difficulty are multiplying polynomials and factoring. Jeffery has a positive attitude and is motivated to learn strategies that will assist him in passing his algebra course. His teacher has identified the following goals for Jeffery:

- Given polynomial problems (e.g., \((x + 5)(x - 3)\); \((4x + 6) + (5x - 8)\), Jeffery will distribute and put into simplest form.
- Given equations (e.g., \(9x^2 + 12\); \(x^2 + 6x + 10\), Jeffery will factor.

POSSIBLE STRATEGIES
- Teaching Vocabulary
- Concrete-Representational-Abstract Method
- Mnemonic Devices

ASSIGNMENT
1. Read the STAR Sheets for the three strategies listed above.
2. Explain how and why each strategy could assist Jeffery in reaching his goals.
3. Explain how you would involve Jeffery’s parents, and develop an activity from one of the strategies that Jeffery’s parents could use at home.
OVERVIEW OF BASIC ALGEBRA SKILLS

- Add, subtract, multiply, and divide integers
- Add, subtract, multiply, and divide equations
- Solve expressions with variables
- Solve two-step equations
- Solve multi-step equations
- Solve real-world algebra problems
- Understand the algebraic order of operations
- Graph coordinates
- Understand functions

BACKGROUND

Student: Cynthia

Age: 14.3

Grade: 9th

SCENARIO

Cynthia is a polite student who has a good attitude regarding school. She enjoys coming to school and has good attendance. She also enjoys working in groups with her peers, and they enjoy working with her. Her teacher notices, however, that she is having difficulty with the higher-level algebra concepts presented. Cynthia tries very hard and has a basic understanding of what to do but has trouble answering the problems on paper when it is time to work by herself. It is the middle of the school year and Cynthia’s teacher is concerned that she will continue having difficulty with the higher-level concepts, and that her positive attitude will turn to a negative one.

AREAS OF STRENGTH

- Proficient in basic facts
- Understands how to manipulate integers
- Can combine like terms

ASSIGNMENT

- Develop three to four goals for Cynthia.
- Using the Algebra STAR Sheets, select a strategy for each goal and explain the benefit of using each strategy to address the corresponding goal.
- Select one goal and describe an independent practice activity that will assist Cynthia in achieving that goal.
WHAT A STAR SHEET IS...

A STAR (Strategies And Resources) Sheet provides you with descriptions of strategies that will help you solve the case studies in this unit.

WHAT IT IS...

Explicit instruction in teaching new vocabulary terms requires the teacher to directly teach the pronunciation and definitions of new vocabulary words in a highly organized manner. In the area of algebra, it also requires teachers to provide students with examples of problem types.

WHAT THE RESEARCH AND RESOURCES SAY...

- Solving word problems is dependent on understanding the language in the word problems.
- To avoid memory gaps and misunderstanding, vocabulary should be explicitly pre-taught and reviewed before each new math lesson.
- Students often require examples illustrating the context of vocabulary words, and they should be encouraged to use the words in journals, presentations, and explanations of work.
- When learning is difficult and novel, teachers must provide support for their students.
- Teachers must model what they want students to learn and provide guided instruction, independent practice, and frequent feedback.
- Students must be given ample opportunities to practice the task in order to generalize the strategy for other settings.

TYPES OF ACTIVITIES TO IMPLEMENT...

- Pre-Teaching Vocabulary
  One easy way to increase students’ vocabulary is to pre-teach vocabulary terms before students encounter the words in their math textbooks or during lectures. When pre-teaching vocabulary, it is important to teach a word within its context. Carnine, Silbert, & Kameenui (1997) suggest the following method for teaching new vocabulary words:
    - State the definitions, and have students repeat the definitions.
    - Provide students with good and bad examples of the words.
    - Review the new words along with previously learned words to ensure students have the words in their long-term memories.

- Self-Correcting Activities
  Once students have explicitly been taught the new vocabulary terms, they can continue practicing the words by using self-correcting word cards. Students are given cards with the vocabulary words written on them and another set of cards with the definitions written on them. Students then match the word with the correct definition. As seen in the example on the next page, students can correct their work by making sure the word and the definition have the same symbol in the upper-right-hand corner. This activity can be completed with a peer, in small groups, or independently.
**Keep in Mind...**

- Students must be taught the vocabulary terms before working on the self-correcting and picture-card activities.
- Pre-teaching vocabulary and self-correcting activities can be used in peer-tutoring settings, small-group settings, or as independent work.
- It is important to provide an example of the definition to which students can relate.
- Although these activities may seem juvenile, students will appreciate having the vocabulary cards for reference as needed.

A list of important vocabulary terms for Case Studies A1, A2, and A3 are provided on the next page.

**Resources...**


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**Variable**

- A letter used to represent one or more numbers in an expression, equation, or inequality.
- Example: \(4b = 12\)
  
  \(b = \text{variable}\)

**Integers**

- The set of whole numbers and their opposites.
- Example: \((1 \text{ and } -1)\)
  
  \((20 \text{ and } -20)\)

**Solution to an Equation**

- A set of values for the variables in an equation that make a true statement when substituted into the equation.
- Example: \(4b = 12\)
  
  \(4(3) = 12\) Substitute \(b = 3\)
  
  \(12 = 12\) True statement
**Vocabulary Terms**

Level A – Case 1

- Absolute Value: the distance from a point on the number line to zero
- Inequality: a mathematical sentence showing quantities are not equal, using <, >, ≤, ≥, or ≠

Level A – Case 2

- Base: a number that is to be multiplied by itself a specified number of times
- Coefficient: the number that is multiplied by the variable in an algebraic expression such as 5b
- Exponent: the number that indicates how many times the base is used as a factor
- Exponential Function: a function where the base is a known value and the exponent is a variable (e.g., \( y=3^x \))
- Monomial: a single term that consists of a number, a variable, or the product of a number and one or more variables (e.g., 7, a, 7b or ab)
- Scientific Notation: a number expressed as a decimal number between 1 and 10 that is then multiplied by a power of 10; used to write extremely large and small numbers

Level A – Case 3

- Binomial: a polynomial with two unlike terms ((2x + 3y) or (3x + 7) or (6x - 18) or (6x^{10} - 8x^4) or \((4ab - 6a^3b^4)\))
- Factorial: the product of all whole numbers, except zero, less than or equal to a number
- Polynomial: a monomial or the sum or difference of two or more monomials ((2x + 3y) or (3x + 2y - 7))
- Trinomial: a polynomial with three unlike terms ((x^2 + x + 1) or (3x^5 - 8x^4 + x^3) or (a^2b + 12x + y))
What a STAR Sheet Is...

A STAR (Strategies And Resources) Sheet provides you with descriptions of research-based strategies that will help you solve the case studies in this unit.

What It Is...

Graphic organizers are visuals that assist students in remembering information. In the area of algebra, graphic organizers may be used to provide students with formulas or cues needed to solve the problems.

What the Research and Resources Say...

• Graphic organizers allow students to see relationships between ideas.
• Few language skills are necessary to understand graphic organizers.
• Teachers can use graphic organizers to diagnose areas of difficulty.
• The strategy can be generalized for use in many situations.
• Graphic organizers allow students to organize information.

Tips for Implementation...

• Graphic organizers should be part of the instruction but not a substitute for it.
• Students must have an understanding of prerequisite skills needed to complete the graphic organizer.
• It is not necessary for students to memorize all procedures, but they need to know where to access the information (i.e., cue cards).
• Graphic organizers should not be complicated: the simpler, the better.

Types of Activities to Implement...

• Cue cards can assist students in remembering the rules.

**Addition and Subtraction Properties of Inequalities**

If \( a < b \), then \( a + c < b + c \).
If \( a > b \), then \( a + c > b + c \).

If \( a < b \), then \( a - c < b - c \).
If \( a > b \), then \( a - c > b - c \).

**Multiplication and Division Properties of Inequalities**

For \( c > 0 \):
If \( a < b \), then \( ac < bc \) AND \( a + c < b + c \).
If \( a > b \), then \( ac > bc \) AND \( a + c > b + c \).

For \( c < 0 \):
If \( a < b \), then \( ac > bc \) AND \( a + c > b + c \).
If \( a > b \), then \( ac < bc \) AND \( a + c < b + c \).
Graphic organizers can be used to assist students in breaking down the steps of problems.

\[
4^6 = 4096
\]

Multiply the base by itself 6 times

- Base
- Exponent

\[
\begin{align*}
4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 &= 16 \cdot 16 \cdot 16 \\
&= 256 \cdot 16 \\
&= 4096
\end{align*}
\]

**Resources...**

What a STAR Sheet Is…

A STAR (Strategies And Resources) Sheet provides you with descriptions of researched-based strategies that will help you solve the case studies in this unit.

What It Is…

The Concrete-Representational-Abstract (CRA) Method of teaching mathematical concepts is a method that allows students to understand a concept before memorizing the algorithms: 1) During the concrete stage, students interact and manipulate three-dimensional objects (e.g., algebra tiles, algebra manipulatives with representations of variables and units). This interaction will assist students in understanding the concept, instead of simply having them solve the algorithm. 2) During the representational stage, students use two-dimensional objects (e.g., pictures) to represent the problems. These pictures may be presented to them by the teacher or through the curriculum used in the class, or students may draw their own representations of the problem. 3) The abstract stage requires students to complete the algorithm without any concrete or representational assistance.

What the Research and Resources Say…

- CRA is effective at all age levels and can assist students in learning basic concepts, operations, and applications.
- Students do not need a large amount of formal experience at the concrete and representational levels to understand the algorithms.
- Students demonstrate a conceptual understanding of the process when using this method, rather than just completing the algorithm.

Tips for Implementation…

- Teachers must be very familiar with the concrete objects prior to teaching and having students interact with them.
- Teachers must provide modeling at all three stages of the CRA method.
- Teachers should continuously monitor student work during the concrete and representational levels, asking them questions about their thinking and providing clarification as needed.

Types of Activities to Implement…

- Algebra manipulatives:
  - Green squares represent 1. The squares have a “+” or a “-” on them.
  - Yellow rectangles represent x. The rectangles have a “+” or a “-” on them.
  - Yellow squares represent $x^2$. The squares have a “+” or a “-” on them.
  - Other manipulatives can be used to show $y$, $y^2$, $y^3$, and $x^3$.

*These manipulatives may only be used to solve equations using whole integers.
*The manipulatives used for the following examples are suggestive (in color and symbolic value) of Algeblocks®, a registered trademark, and Algebra Tiles™.

Note: Teachers must have a good understanding of manipulatives and their uses prior to engaging students in their use.
**Multiplying Polynomials**

1A.

\[(x + 4) (x - 2)\]

**Step 1:** Write out the problem

\[(x + 4) (x - 2)\]

**Step 2:** Using algebra manipulatives, show 

\[(x + 4) (x - 2)\]

**Step 3:** Multiply all factors and fill in the grid

**FACTOR DETAIL:**
Step 4: Cross out equal pairs

Step 5: Write out the answer

\[(x + 4)(x - 2) = x^2 + 2x - 8\]
Factoring Polynomials

1B.

\[ x^2 + 4x + 4 \]

Step 1: Write out the problem

Step 2: Using algebra manipulatives, show \( x^2 + 4x + 4 \) (must be in a perfect square or rectangle)

Step 3: Factor out the problem

Step 4: Write out the answer

\[ x^2 + 4x + 4 = (x + 2)(x + 2) \]
**Keep in Mind...**

- Activities during the concrete and representational stages must represent the actual process so that students are able to generalize the process during the abstract stage.
- Students must be able to manipulate the concrete objects; therefore, you must have enough objects for students to use either individually or in small groups (comprised of no more than three students).

**Resources...**


WHAT A STAR SHEET IS...
A STAR (Strategies And Resources) Sheet provides you with descriptions of research-based strategies that will help you solve the case studies in this unit.

WHAT IT IS...
Mnemonic devices are words, sentences, or pictorial devices created to help students remember content.

WHAT THE RESEARCH AND RESOURCES SAY...
• Mnemonics can assist students in acquiring and retaining information.
• Mnemonics are effective for learning a new process.
• Mnemonics can be developed by either the teacher or the student.

TIPS FOR IMPLEMENTATION...
• Strategies must be modeled by the teacher prior to student use.
• Mnemonics must be appropriate and match the content being presented.
• The use of mnemonics must be explicitly taught.
• Ample practice is needed for students to independently use the mnemonics.

TYPES OF ACTIVITIES TO IMPLEMENT...
• FOIL Strategy
  FOIL, a problem-solving strategy for multiplying two binomials, stands for “First, Outside, Inside, Last.” Teachers may want to consider using a mnemonic device like FOIL if students are having difficulty multiplying binomials. The figure below illustrates the guidelines students should follow to apply FOIL to algebra problems.

![FOIL Diagram]

• Please Excuse My Dear Aunt Sally (parentheses, exponents, multiply, divide, add, subtract)
  This strategy should be taught to students who are having difficulty with the order of operations needed in solving algebra problems. The mnemonic provides the sequence of steps needed.

RESOURCES...